

NATIONAL UNIVERSITY OF SINGAPORE
 Department of Mathematics
 MA 1505 Mathematics I
 Tutorial 9

1. A table of values of a function f is given below.

$x \backslash y$	0	1	2
0	1	6	4
1	3	5	7
2	8	2	9

For example, $f(1, 1) = 5$. Find $\int_C \nabla f \bullet dr$, where

- (a) C has parametric equations $x = t^2 + 1, y = t^3 + t, 0 \leq t \leq 1$.
 (b) C is the unit circle $x^2 + y^2 = 1$.

Ans: (a) 6; (b) 0

2. The base of a circular fence with radius 10 m is given by $x = 10 \cos t, y = 10 \sin t$. The height of the fence varies from 3 m to 5 m such that, at position (x, y) , the height is given by the function $h(x, y) = 4 + 0.01(x^2 - y^2)$. Suppose that 1 litre of paint covers 100 m^2 . Sketch the fence and determine how much paint you will need if you paint both sides of the fence.

Ans: 5 litre

3. An 80 kg man carries a pail of water weighing 10 kg up a helical (spiralling) staircase with radius 6 m. The man makes exactly three complete revolutions and reaches a height of 30 m. Suppose a total of 2 kg of water leaks steadily during the man's ascent. Express the work done by the man against gravity $g \text{ ms}^{-2}$ as a line integral $\int_C \mathbf{F} \bullet d\mathbf{r}$ by writing down the vector field \mathbf{F} and the vector equation of the curve C .

Evaluate the work done by computing this line integral.

Ans: $2670g \text{ kg}\cdot\text{m}^2\text{s}^{-2}$

4. Evaluate the line integral $\oint_C xy^2 dx + x^3 dy$, where C is the rectangle with vertices $(0, 0), (2, 0), (2, 3)$ and $(0, 3)$ oriented anticlockwise by (i) direct computation, and (ii) using Green's Theorem.

Ans: 6

5. Evaluate $\oint_C (x^5 - y^5) dx + (x^5 + y^5) dy$, where C is the boundary with positive orientation of the region between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$, where $0 < a < b$.

Ans: $5\pi(b^6 - a^6)/4$