950/1, 954/1 **STPM (Trial) 2011**

MATHEMATICS S
MATHEMATICS T

PAPER 1

Three hours

PEPERIKSAAN PERCUBAAN
SIJIL TINGGI PERSEKOLAHAN MALAYSIA
NEGERI SEMBILAN DARUL KHUSUS 2011

Instruction to candidates:

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Answer **all** questions. Answer may be written in either English or Bahasa Malaysia.
All necessary working should be shown clearly.
Non-exact numerical answer may be given correct to three significant figures, or one decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Mathematical tables, list of mathematical formulae and graph paper are provided.

Arahan kepada calon:

JANGAN BUKA KERTAS SOALAN INI SEHINGGA ANDA DIBENARKAN BERBUAT DEMIKIAN.

Jawab **semua** soalan. Jawapan boleh ditulis dalam Bahasa Inggeris atau Bahasa Malaysia.
Semua kerja yang perlu hendaklah ditunjukkan dengan jelas.
Jawapan berangka tak tepat boleh diberikan betul sehingga tiga angka bererti, atau satu tempat perpuluhan dalam kes sudut dalam darjah, kecuali aras kejituan yang lain ditentukan dalam soalan.
Sifir matematik, senarai rumus matematik, dan kertas graf dibekalkan.

This question paper consists of 3 printed pages.
(Kertas soalan ini terdiri daripada 3 halaman)

1. Express $\frac{x^2-3x}{x^2-2x+1}$ in partial fractions. [5 *marks*]

2. By using the substitution x = 1 + u, find the exact value of $\int_2^5 \frac{4x^2}{(x-1)^2}\,dx$. [5 *marks*]

3. If $|z| = \sqrt{5}$, find the modulus of $z + \frac{1}{z^*}$, where z^* is the complex conjugate of z.

[6 *marks*]

4. Given the polynomial $f(x) = x^4 - 3x^3 + kx^2 + 15x + 50$, where k is a constant and that $(x - 5)$ is a factor of f(x). Find the value of k and hence factorise f(x) completely into exact linear factors. [6 *marks*]

5. Use the binomial theorem to expand $\sqrt{\frac{4+x}{1-x}}$ as a series of ascending powers of x up to and including the term in x^2. Find the set of values of x such that the expansion is valid. [7 *marks*]

6. Given that function $f(x) = \begin{cases} |1+x|, & if\ x \le 0 \\ x^2, & if\ x > 0 \end{cases}$

   (a) Sketch the graph of $y = f(x)$. State the range of $f(x)$.

   (b) Hence, find the set of values of x for which $f(x) - 1 \ge 0$. [7 *marks*]

7. Find the set of values of x which satisfies $\frac{2}{x} + 1 \ge 4 - \frac{1}{2-x}$. [7 *marks*]

8. Given that matrices $\mathbf{A} = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 1 & 3 \\ 4 & 2 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -5 & 0 & 5 \\ 10 & -1 & -7 \\ 0 & 2 & -1 \end{pmatrix}$.

   (a) Show that **A** is a non-singular matrix.

   (b) Find matrix **AB** and deduce $\mathbf{A}^{-1}$.

   (c) Given matrix $\mathbf{C} = \begin{pmatrix} n \\ n \\ n \end{pmatrix}$, find matrix **X** in terms of n if **AX** = **C**. [9 *marks*]

9. A circle touches the straight line $4y = 3x - 8$ at the point $P(4, 1)$ and passes through another point $Q(5, 3)$. Find the equation of the circle and show that it touches the y-axis. [10 *marks*]

10. Find the coordinates of the stationary points on the curve $y = e^{-x}x^2$ and determine their nature. Sketch the curve. [11 *marks*]

11. Sketch on the same coordinate axes, the curve $y = 2x^2$ and the ellipse $x^2 + \frac{y^2}{9} = 1$.

(a) Calculate the area of the region bounded by the curve $y = 2x^2$ and the line $y = 1\frac{1}{2}$.

(b) Find the volume of the solid formed when the region bounded by the curve and the ellipse is rotated through 180° about the y-axis. [12 *marks*]

12. (a) Express $\frac{1}{(3k-2)(3k+1)}$ in partial fractions. Hence, obtain an expression for

$S_n = \sum_{k=1}^{n} \frac{1}{(3k-2)(3k+1)}$ and find $\lim_{n\to\infty} S_n$. [8 *marks*]

(b) Find the least value of n for which the sum of the first n terms of the geometric series
$1 + 0.75 + (0.75)^2 + (0.75)^3 + \ldots$
is greater than $\frac{3}{4}$ of its sum to infinity. [7 *marks*]

954/2 STPM 2011

MATHEMATICS T (MATEMATIK T)

PAPER 2 (PAPER 2)

Three hours (tiga jam)

**PEPERIKSAAN PERCUBAAN
SIJIL TINGGI PERSEKOLAHAN MALAYSIA
NEGERI SEMBILAN DARUL KHUSUS 2011**

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Mathematical tables, list of mathematical formulae and graph paper are provided.

Arahan kepada calon:

JANGAN BUKA KERTAS SOALAN INI SEHINGGA ANDA DIBENARKAN BERBUAT DEMIKIAN.
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Sifir matematik, senarai rumus matematik, dan kertas graf dibekalkan.

This question paper consists of 4 printed pages.
(Kertas soalan ini terdiri daripada 4 halaman bercetak)

1. Express 12 cos θ + 5 sin θ in the form r cos(θ - α), where r > 0 and $0^0 < \alpha < 90^0$. [3 *marks*]

   Hence, solve 12 cos θ + 5 sin θ = 3 for $0^0 < \theta < 360^0$. [3 *marks*]

2. By using the substitution $\sin\theta = \dfrac{2t}{1+t^2}$ and $\cos\theta = \dfrac{1-t^2}{1+t^2}$, where $t = \tan\dfrac{\theta}{2}$, show that

$$\frac{1-\sin\theta}{1+\sin\theta} = \tan^2\left(\frac{1}{4}\pi - \frac{1}{2}\theta\right)$$ [3 *marks*]

   Hence, or otherwise, by using the substitution $\theta = \dfrac{1}{4}\pi$, show that $\tan\left(22\tfrac{1}{2}^0\right) = \sqrt{2} - 1$.

   [4 *marks*]

3. By using the substitution $y = ux^2$, show that the differential equation

   $x^2\dfrac{dy}{dx} - 2xy + 3 = 0$ may be reduced to $\dfrac{du}{dx} = -\dfrac{3}{x^4}$. [3 *marks*]

   Hence, show that the general solution for *y* in terms of *x* is $y = Cx^2 + \dfrac{1}{x}$, where C is an arbitrary constant. [3 *marks*]

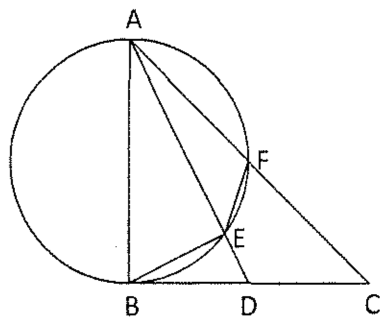
4. The position vectors of points A, B and C are $9\underset{\sim}{i} - 10\underset{\sim}{j}$, $4\underset{\sim}{i} + 2\underset{\sim}{j}$, and $k\underset{\sim}{i} - 2\underset{\sim}{j}$ respectively.

   (a) Find the value of *k* if the points *A, B* and *C* are collinear. [4 *marks*]

   (b) If *ABCD* is a parallelogram, show that the position vector of point D is $-9\underset{\sim}{i} - 14\underset{\sim}{j}$.

   [4 *marks*]

5.



In the above diagram, *AB* is the diameter of the circle *ABEF* and *BC* is a tangent to the circle at *B*. *AED*, *AFC* and *BDC* are straight lines.

   (a) Prove that *CDEF* is a cyclic quadrilateral. [5 *marks*]

   (b) Show that ΔAEF and ΔACD are similar and hence show that *AF.AC* = *AE.AD*

   [5 *marks*]

6. Two rods PQ and PR, of fixed lengths r cm and q cm respectively, are hinged together at P and $\angle QPR = \theta$. Initially, $\theta = \frac{\pi}{6}$ and θ varies such that the rate of decrease of the area, A, of the triangle PQR with respect to time, t, is $3qr \sin\theta$ cm^2 s^{-1}.

   (a) State the area A of the triangle PQR and show that $\frac{dA}{d\theta} = \frac{1}{2} qr\cos\theta$ [2 *marks*]

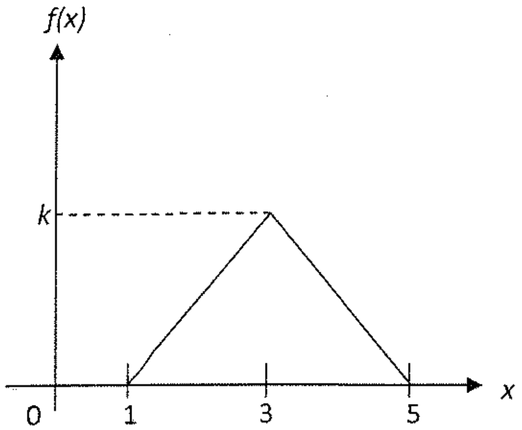
   (b) Show that $\frac{d\theta}{dt} = -6\tan\theta$. [3 *marks*]

   Hence, or otherwise, determine the particular solution of the equation $\frac{d\theta}{dt} = -6\tan\theta$. [6 *marks*]

   (c) Find the area of the triangle PQR, in terms of q, r, when t = 0.5 sec. [3 *marks*]

7. An infectious flu virus is spreading through a school. The probability of a student in the school having the flu is 0.1. If four or more of the students out of a class of 35 students are away with the flu, a class test will have to be cancelled. By using a suitable approximation, find the probability that the test will be cancelled. [4 *marks*]

8. The graph of the probability density function of a continuous random variable X is given below.



   (a) Determine the value of k [2 *marks*]

   (b) Find the probability density function, $f(x)$ of X. [4 *marks*]

9. A three digit number is formed without repetition from the set of integers $\{1, 2, 3, 4, 5, 6\}$. Events A and B are defined as follows :

   A : the number does not contain the digit 6.

   B : the number begins with the digit 3.

   (a) Find P(A) and P(B). [3 *marks*]

   (b) Find the conditional probability of event A given that event B has occurred. [3 *marks*]

   (c) State, with reason, whether events A and B are independent. [2 *marks*]

10. A random sample of 25 people was asked to record the number of kilometers they travelled by car in a given week. The distances, to the nearest km, are shown below.

48	52	66	54	80
47	81	53	63	81
50	47	67	74	76
82	46	58	68	77
62	63	68	72	78

(a) Display the above data in an ordered stemplot. [2 *marks*]
(b) Find the median and the interquartile range. [3 *marks*]
(c) Draw a boxplot to represent the above data. [3 *marks*]
(d) Find the mean and standard deviation of this data. [5 *marks*]
(e) State with reason the shape of the frequency distribution. [2 *marks*]

11. The discrete random variable X has the probability distribution function

$P(X = x) = k\left|\frac{3}{2} - x\right|$, $x = 1, 2, 3$ where k is a constant.

(a) Find the value of k [2 *marks*]
(b) Show that $E(X) = \frac{12}{5}$, and find $Var(X)$ [5 *marks*]

12. A machine in a factory produces metal rods. The diameter of the rods produced are normally distributed with mean 2.00 cm and standard deviation σ cm.

Another machine in the factory produces metal rings. The internal diameter of the rings produced are normally distributed with mean (2.00 + 3σ) cm and standard deviation 3σ cm.

Find the probability that a randomly chosen ring can be threaded on a randomly chosen rod. [5 *marks*]

A rod is oversized if its diameter exceeds 2.05 cm. It is found that 1% of the rods produced by the machine are oversized. Find the value of σ. [4 *marks*]

Skema Pemarkahan Mathematics T1/S1 Peperiksaan Percubaan STPM 2011

1. Express $\frac{x^2-3x}{x^2-2x+1}$ in partial fractions. [5 *marks*]

$\frac{x^2-3x}{x^2-2x+1} \equiv 1-\frac{x+1}{x^2-2x+1}$ M1

Let $\frac{x+1}{(x-1)(x-1)} \equiv \frac{A}{x-1}+\frac{B}{(x-1)^2}$ B1

$x+1 = A(x-1)+B$ M1

$x=1, B=2;\ x=0, A=1$ M1

$\therefore \frac{x^2-3x}{x^2-2x+1} \equiv 1-\frac{1}{x-1}-\frac{2}{(x-1)^2}$ A1 [5]

2. By using the substitution $x = 1 + u$, find the exact value of $\int_2^5 \frac{4x^2}{(x-1)^2}dx$. [5 *marks*]

$x = 1+u,\ u = x-1,\ du = dx,$
$x = 2, u = 1;\ x = 5, u = 4$ B1

$\int_2^5 \frac{4x^2}{(x-1)^2}dx = \int_1^4 \frac{4(1+u)^2}{u^2}du$ M1

$= 4\int_1^4 \left(\frac{1}{u^2}+\frac{2}{u}+1\right)du$

$= 4\left[-\frac{1}{u}+2\ln u+u\right]_1^4$ A1

$= 4\left[\left(-\frac{1}{4}+2\ln 4+4\right)-\left(-1+2\ln 1+1\right)\right]$ M1

$= 15+16\ln 2$ A1 [5]

3. If $|z| = \sqrt{5}$, find the modulus of $z + \frac{1}{z^*}$, where z^* is the complex conjugate of z.

[6 *marks*]

Let $z = a + bi$

$|z| = \sqrt{a^2 + b^2} = \sqrt{5}$ B1

$z + \frac{1}{z^*} = a + bi + \frac{1}{a - bi}$

$= a + bi + \frac{a + bi}{a^2 + b^2}$ M1

$= a + bi + \frac{1}{5}(a + bi)$ M1

$= \frac{6}{5}(a + bi)$ A1

$\left|z + \frac{1}{z^*}\right| = \sqrt{\left(\frac{6}{5}a\right)^2 + \left(\frac{6}{5}b\right)^2}$ M1

$= \frac{6}{5}\sqrt{a^2 + b^2} = \frac{6}{5}\sqrt{5}$ A1 [6]

4. Given the polynomial $f(x) = x^4 - 3x^3 + kx^2 + 15x + 50$, where k is a constant and that $(x - 5)$ is a factor of f(x). Find the value of k and hence factorise f(x) completely into exact linear factors. [6 *marks*]

$f(5) = 0 \Rightarrow (5)^4 - 3(5)^3 + k(5)^2 + 15(5) + 50 = 0$ M1

$\Rightarrow k = -15$ A1

$f(-2) = (-2)^4 - 3(-2)^3 - 15(-2)^2 + 15(-2) + 50 = 0$ M1

$\therefore x + 2$ is a factor of f(x) A1

$f(x) = (x + 2)(x - 5)(x^2 - 5)$ M1

$= (x + 2)(x - 5)(x - \sqrt{5})(x + \sqrt{5})$ A1 [6]

5. Use the binomial theorem to expand $\sqrt{\frac{4 + x}{1 - x}}$ as a series of ascending powers of x up to and including the term in x^2. Find the set of values of x such that the expansion is valid. [7 *marks*]

$(4 + x)^{\frac{1}{2}} = 2\left(1 + \frac{x}{4}\right)^{\frac{1}{2}}$

$= 2\left[1 + \frac{1}{2}\left(\frac{x}{4}\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{x}{4}\right)^2 + \ldots\right]$ M1

$= 2 + \frac{1}{4}x - \frac{1}{64}x^2 + \ldots$ A1

$$(1-x)^{-\frac{1}{2}} = 1+\left(-\frac{1}{2}\right)(-x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(-x)^2+\ldots$$ M1

$$= 1+\frac{1}{2}x+\frac{3}{8}x^2+\ldots$$ A1

$$\sqrt{\frac{4+x}{1-x}} \approx \left[2+\frac{1}{4}x-\frac{1}{64}x^2\right]\left[1+\frac{1}{2}x+\frac{3}{8}x^2\right]$$ M1

$$\approx 2+\frac{5}{4}x+\frac{55}{64}x^2$$ A1

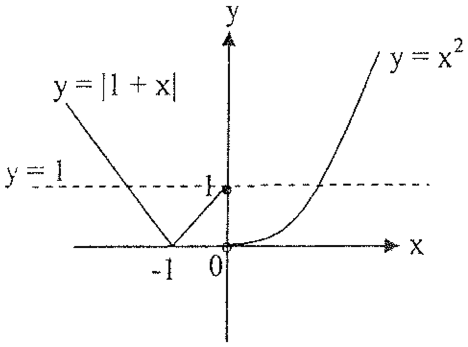
∴ Set of values of x for the expansion to be valid is { x: -1 < x < 1} A1 [7]

6. Given that function $f(x)=\begin{cases}|1+x|, & if\ x \le 0\\ x^2, & if\ x>0\end{cases}$

(a) Sketch the graph of $y=f(x)$. State the range of $f(x)$.

(b) Hence, find the set of values of x for which $f(x)-1 \ge 0$. [7 *marks*]

(a)



D1 $y = |1 + x|$

D1 $y = x^2$

D1 all correct

Range of f = {y: y ≥ 0} B1

$f(x) - 1 \ge 0 \quad \Rightarrow f(x) \ge 1$

Solving y = 1 and y = -(x + 1) ⇒ x = -2 M1

Solving y = 1 and $y = x^2$ ⇒ x = 1 M1

The set of values of x is {x : x ≤ -2 or x ≥ 1 or x = 0} A1 [7]

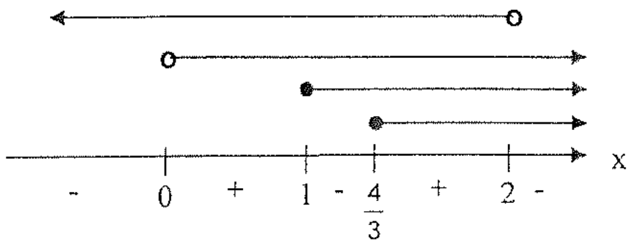
7. Find the set of values of x which satisfies $\frac{2}{x} + 1 \geq 4 - \frac{1}{2-x}$. [7 *marks*]

$\frac{2}{x} + \frac{1}{2-x} - 3 \geq 0$ M1

$\frac{2(2-x) + x - 3x(2-x)}{x(2-x)} \geq 0$ M1

$\frac{3x^2 - 7x + 4}{x(2-x)} \geq 0$ M1

$\frac{(3x-4)(x-1)}{x(2-x)} \geq 0$ M1



M1

The solution set is $\{ x : 0 < x \leq 1 \text{ or } \frac{4}{3} \leq x < 2 \}$ A1 A1 [7]

8. Given that matrices $\mathbf{A} = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 1 & 3 \\ 4 & 2 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -5 & 0 & 5 \\ 10 & -1 & -7 \\ 0 & 2 & -1 \end{pmatrix}$.

(a) Show that **A** is a non-singular matrix.

(b) Find matrix **AB** and deduce $\mathbf{A}^{-1}$.

(c) Given matrix $\mathbf{C} = \begin{pmatrix} n \\ n \\ n \end{pmatrix}$, find matrix **X** in terms of n if **AX** = **C**. [9 *marks*]

(a) $\det \mathbf{A} = 3(1-6) - 2(2-12) + 1(4-4) = 5$ M1

Since $|\mathbf{A}| \neq 0$, $\therefore$ **A** is a non-singular matrix. A1

(b) $\mathbf{AB} = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} -5 & 0 & 5 \\ 10 & -1 & -7 \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ M1 A1

$\mathbf{A}^{-1} = \frac{1}{5}\mathbf{B}$ M1

$= \begin{bmatrix} -1 & 0 & 1 \\ 2 & -\frac{1}{5} & -\frac{7}{5} \\ 0 & \frac{2}{5} & -\frac{1}{5} \end{bmatrix}$ A1

(c) $\mathbf{X} = \mathbf{A}^{-1}\mathbf{C}$ M1

$= \begin{bmatrix} -1 & 0 & 1 \\ 2 & -\frac{1}{5} & -\frac{7}{5} \\ 0 & \frac{2}{5} & -\frac{1}{5} \end{bmatrix} \begin{bmatrix} n \\ n \\ n \end{bmatrix}$ M1

$\begin{pmatrix} 0 \\ \frac{2}{5}n \\ \frac{1}{5}n \end{pmatrix}$ A1 [9]

9. A circle touches the straight line $4y = 3x - 8$ at the point P(4, 1) and passes through another point Q(5, 3). Find the equation of the circle and show that it touches the y-axis. [10 *marks*]

Let the centre of the circle = C (a, b)

CP = CQ

$\Rightarrow \sqrt{(a-4)^2 + (b-1)^2} = \sqrt{(a-5)^2 + (b-3)^2}$ M1

$\Rightarrow 2a + 4b = 17$ ....(1) A1

Equation of CP $\Rightarrow y - 1 = -\frac{4}{3}(x-4)$ M1

$\Rightarrow 3y + 4x = 19$

At C $\Rightarrow 4a + 3b = 19$..................(2) A1

Solving (1) & (2), M1

$a = \frac{5}{2}$, $b = 3$, or $C\left(\frac{5}{2}, 3\right)$ A1

Radius of the circle, $r = \sqrt{\left(4 - \frac{5}{2}\right)^2 + (1-3)^2} = \frac{5}{2}$ A1

Equation of the circle, $(y-3)^2 + \left(x - \frac{5}{2}\right)^2 = \frac{25}{4}$ A1

At y-axis, $x = 0$, $(y-3)^2 + \left(-\frac{5}{2}\right)^2 = \frac{25}{4} \Rightarrow (y-3)^2 = 0$ M1

2 equal roots, the circle touches y-axis. A1 [10]

10. Find the coordinates of the stationary points on the curve $y = e^{-x}x^2$ and determine their nature. Sketch the curve. [11 *marks*]

$\frac{dy}{dx} = 2xe^{-x} - e^{-x}x^2$ M1

$= xe^{-x}(2 - x)$

Let $\frac{dy}{dx} = 0$, $xe^{-x}(2 - x) = 0$ M1

$x = 0$ or $x = 2$

$y = 0$ or $y = \frac{4}{e^2}$ M1

$\therefore$ the stationary points are (0, 0) and $(2, \frac{4}{e^2})$ A1

$\frac{d^2y}{dx^2} = e^{-x}(2 - 2x) - e^{-x}(2x - x^2)$ M1

$= e^{-x}(2 - 4x + x^2)$

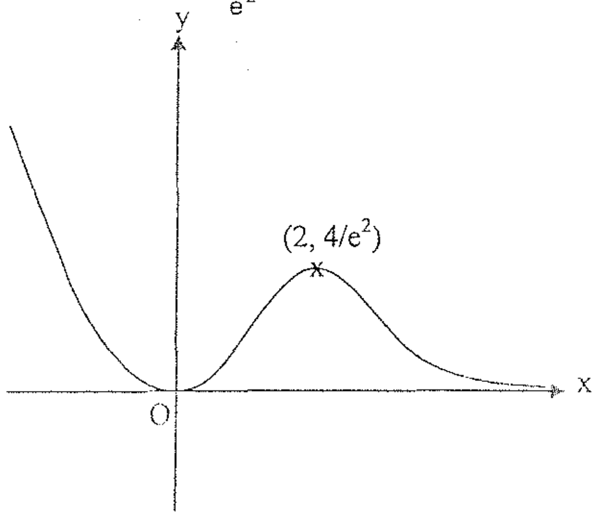
$x = 0$, $\frac{d^2y}{dx^2} = 2 > 0$

$x = 2$, $\frac{d^2y}{dx^2} = -\frac{2}{e^2} < 0$ M1

$\therefore$ (0, 0) is a minimum point and $(2, \frac{4}{e^2})$ is a maximum point. A1 A1

$x \to +\infty, y \to 0$

$x \to -\infty, y \to +\infty$



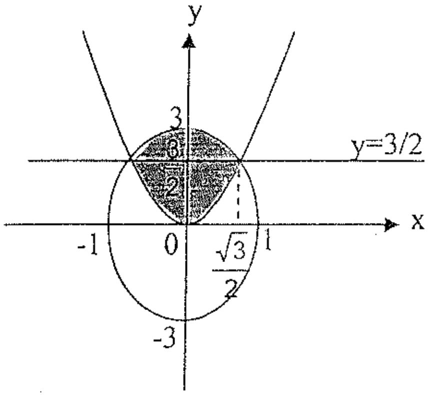
D1 (shape)
D1 (critical points)
D1 (all correct)

[11]

11. Sketch on the same coordinate axes, the curve $y = 2x^2$ and the ellipse $x^2 + \frac{y^2}{9} = 1$.

(a) Calculate the area of the region bounded by the curve $y = 2x^2$ and the line $y = 1\frac{1}{2}$.

(b) Find the volume of the solid formed when the region bounded by the curve and the ellipse is rotated through 180° about the y-axis. [12 *marks*]



$x^2 + \frac{4x^4}{9} = 1 \Rightarrow x = \pm\frac{\sqrt{3}}{2} \Rightarrow y = \frac{3}{2}$ M1 A1

D1 (parabola)

D1(ellipse)

(a) $x = \frac{y^{\frac{1}{2}}}{\sqrt{2}}$

$\text{Area} = 2\int_0^{\frac{3}{2}} \frac{1}{\sqrt{2}}\left(y^{\frac{1}{2}}\right)dy$ M1

$= \frac{2}{\sqrt{2}}\left[\frac{2}{3}Y^{\frac{3}{2}}\right]_0^{\frac{3}{2}}$ M1

$= \frac{4}{3\sqrt{2}}\left(\frac{3}{2}\sqrt{\frac{3}{2}} - 0\right)$ M1

$= \sqrt{3}$ A1

(b) $\text{Volume} = \int_0^{\frac{3}{2}} \pi\left(\frac{1}{2}y\right)dy + \int_{\frac{3}{2}}^{3} \pi\left(1 - \frac{y^2}{9}\right)dy$ M1

$= \left[\frac{1}{4}\pi y^2\right]_0^{\frac{3}{2}} + \left[\pi\left(y - \frac{y^3}{27}\right)\right]_{\frac{3}{2}}^{3}$ M1

$= \pi\left[\left(\frac{9}{16} - 0\right) + \left(3 - 1 - \frac{3}{2} + \left(\frac{3}{2}\right)^3 \div 27\right)\right]$ M1

$= \frac{19}{16}\pi$ A1 [12]

12. (a) Express $\frac{1}{(3k-2)(3k+1)}$ in partial fractions. Hence, obtain an expression for $S_n = \sum_{k=1}^{n} \frac{1}{(3k-2)(3k+1)}$ and find $\lim_{n\to\infty} S_n$. [8 *marks*]

(b) Find the least value of n for which the sum of the first n terms of the geometric series
$1 + 0.75 + (0.75)^2 + (0.75)^3 + \ldots$
is greater than $\frac{3}{4}$ of its sum to infinity. [7 *marks*]

(a) Let $\frac{1}{(3k-2)(3k+1)} \equiv \frac{A}{3k-2} + \frac{B}{3k+1}$ M1

$\Rightarrow 1 = A(3k+1) + B(3k-2)$ M1

$\frac{1}{(3k-2)(3k+1)} \equiv \frac{1}{3(3k-2)} - \frac{1}{3(3k+1)}$ A1

$S_n = \sum_{k=1}^{n} \frac{1}{(3k-2)(3k+1)}$

$= \sum_{k=1}^{n} \frac{1}{3}\left[\frac{1}{3k-2} - \frac{1}{3k+1}\right]$ M1

$= \frac{1}{3}\left[\left(1-\frac{1}{4}\right) + \left(\frac{1}{4}-\frac{1}{7}\right) + \left(\frac{1}{7}-\frac{1}{10}\right) + \ldots\left(\frac{1}{3n-5}-\frac{1}{3n-2}\right) + \left(\frac{1}{3n-2}-\frac{1}{3n+1}\right)\right]$ M1

$= \frac{1}{3}\left[1-\frac{1}{3n+1}\right] = \frac{n}{3n+1}$ A1

$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left[\frac{1}{3}\left(1-\frac{1}{3n+1}\right)\right] = \frac{1}{3}$ M1 A1 [8]

(b) $S_n = \frac{1(1-0.75^n)}{1-0.75}$ M1

$= \frac{1-0.75^n}{0.25}$ A1

$S_\infty = \frac{a}{1-r} = \frac{1}{1-0.75}$ M1

$= \frac{1}{0.25} = 4$ A1

$S_n > \frac{3}{4}(4) \Rightarrow \frac{1-0.75^n}{0.25} > 3$ M1

$\Rightarrow 1 - 0.75^n > 0.75$

$\Rightarrow 0.75^n < 0.25$

$\Rightarrow n > 4.819$ M1

∴ The least value of n is 5. A1 [7]

MARKING SCHEME FOR MATHEMATICS T PAPER 2 TRIAL 2011

1. Let 12 cos θ + 5 sin θ = r cos(θ - α)

$\Rightarrow$ 12 cos θ + 5 sin θ = r cos θ cos α + r sin θ sin α

r cos α = 12 .....(1)

r sin α = 5 ......(2) B1

Squaring (1) and (2) and adding gives:

$r^2 = 169 \Rightarrow r = 13$ M1 (for finding either r or α)

Dividing (2) by (1) gives:

$\tan\alpha = \frac{5}{12} \Rightarrow \alpha = 22.6^0$

$\therefore$ 12 cos θ + 5 sin θ = 13 cos(θ – 22.6^0) A1 (Strict)

12 cos θ + 5 sin θ = 3

13 cos(θ – 22.6^0) = 3 B1

$\cos(\theta - 22.6^0) = \frac{3}{13}$ M1

$\theta - 22.6^0 = 76.7^0, 283.3^0$

$\therefore\ \theta = 99.3^0,\ 305.9^0$ A1

Total = 6 marks

2. Let $t = \tan\frac{\theta}{2}$ then $\sin\theta = \frac{2t}{1+t^2}$ and $\cos\theta = \frac{1-t^2}{1+t^2}$

$LHS = \frac{1-\sin\theta}{1+\sin\theta}$

$= \frac{1-\frac{2t}{1+t^2}}{1+\frac{2t}{1+t^2}}$ B1

$= \frac{1+t^2-2t}{1+t^2+2t}$

$= \frac{(1-t)^2}{(1+t)^2}$

$= \left(\frac{\tan\frac{1}{4}\pi - \tan\frac{1}{2}\theta}{1+\tan\frac{1}{4}\pi\tan\frac{1}{2}\theta}\right)^2$ B1 (for $\tan\frac{\pi}{4}$)

B1 (for changing to the required form)

$= \tan^2\left(\frac{1}{4}\pi - \frac{1}{2}\theta\right) = RHS$

Substituting into the equation

$$\tan^2 22\frac{1}{2}^0 = \frac{1-\sin\frac{1}{4}\pi}{1+\sin\frac{1}{4}\pi}$$ M1

$$= \frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}-1}{\sqrt{2}+1}$$ B1 (for substituting $\frac{1}{\sqrt{2}}$)

$$= \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$ M1 (for rationalizing the denominator)

$$= \frac{\left(\sqrt{2}-1\right)^2}{1} = \left(\sqrt{2}-1\right)^2$$

$$\therefore \tan 22\frac{1}{2}^0 = \sqrt{2}-1$$ A1

Total = 7 marks

3. $\frac{d}{dx}(ux^2) = 2ux + x^2\frac{du}{dx}$ B1

$x^2\,(2ux + x^2\frac{du}{dx}) - 2x\,(ux^2) + 3 = 0$ M1 (his $\frac{d}{dx}(ux^2)$)

$\frac{du}{dx} = -\frac{3}{x^4}$ A1

$\int du = -\int \frac{3}{x^4}\,dx$ B1 (separable variables)

$u = x^{-3} + C$ M1

$y = Cx^2 + \frac{1}{x}$ A1

Total : 6 marks

4. A, B and C are collinear

$\overrightarrow{AB} = \lambda \overrightarrow{BC}$

$-5\underset{\sim}{i} + 12\underset{\sim}{j} = \lambda(\overrightarrow{OC} - \overrightarrow{OB})$ M1 (his $\overrightarrow{AB}$ or $\overrightarrow{BC}$)

$-5\underset{\sim}{i} + 12\underset{\sim}{j} = \lambda[(k\underset{\sim}{i} - 2\underset{\sim}{j}) - (4\underset{\sim}{i} + 2\underset{\sim}{j})]$

$\begin{pmatrix} -5 \\ 12 \end{pmatrix} = \lambda \begin{pmatrix} k-4 \\ -4 \end{pmatrix}$

$-5 = \lambda(k-4)$ .... (1)

$12 = -4\lambda$ .... (2) B1

From (2) ,

$12 = -4\lambda$

$\lambda = -3$ M1 (for solving λ)

Substitute $\lambda = -3$ into (1)

Therefore $k = 5\frac{2}{3}$ A1

(b) Let the position vector of point D be $\overrightarrow{OD} = h\underset{\sim}{i} + k\underset{\sim}{j}$

Since ABCD is a parallelogram

Therefore

$\overrightarrow{AB} = \overrightarrow{DC}$, $\overrightarrow{BC} = \overrightarrow{AD}$ B1 (either one)

$\overrightarrow{AB} = \overrightarrow{OC} - \overrightarrow{OD}$

$\begin{pmatrix} -5 \\ 12 \end{pmatrix} = \begin{pmatrix} k \\ -2 \end{pmatrix} - \begin{pmatrix} h \\ k \end{pmatrix}$ M1

$\begin{pmatrix} -5 \\ 12 \end{pmatrix} = \begin{pmatrix} k-h \\ -2-k \end{pmatrix}$

$12 = -2 - k$ ......(1) M1 (for giving equations 1 and 2)

$k = -14$

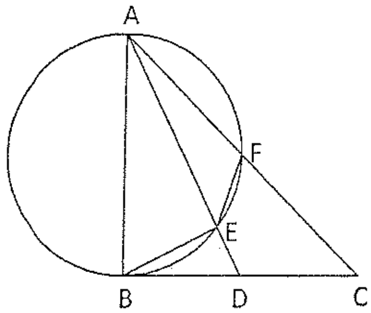
$-5 = k - h$ ....(2)

$h = -9$

$\overrightarrow{OD} = -9\underset{\sim}{i} - 14\underset{\sim}{j}$ A1

Total = 8 marks

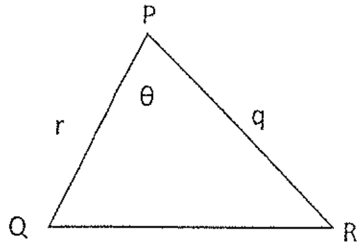
5.



(a) $\angle BEA = 90^\circ$ (angle sub by diameter)	B1	
Let $\angle BAE = \alpha$ $\angle EBD = \angle BAE$ (alternate segment) $= \alpha$	B1	
$\angle EDC = 90^\circ + \alpha$ (exterior angle of ΔBDE)	B1	
$\angle ABE = 90^\circ - \alpha$ (remaining angle of ΔABE)		
$\angle AFE = 180^\circ - \angle ABE$ (opposite angles of c.q) $= 90^\circ + \alpha$	B1	
$\therefore \angle AFE = \angle EDC$ $CDEF$ is a cyclic quadrilateral. (exterior angle of c.q)	B1	(5)

(b) $\angle AFE = \angle EDC = \angle ADC$ (exterior angle of c.q)	B1	
$\angle EAF = \angle DAC$ (common angle)	B1	
$\angle AEF = \angle ACD$ (remaining angle)		
$\therefore \Delta AEF$ and ΔACD are similar.	B1	
$\dfrac{AF}{AD} = \dfrac{AE}{AC}$	B1	
$AF.AC = AE.AD$	B1	(5)

6.



(a) Area of ΔPQR,

$A = ½\ qr \sin\theta$ B1

$\frac{dA}{d\theta} = \frac{1}{2}qr\cos\theta$ B1

(b) Given $\frac{dA}{dt} = -3qr\sin\theta$ B1

$\frac{d\theta}{dt} = \frac{d\theta}{dA} \times \frac{dA}{dt}$ M1

$= \frac{2}{qr\cos\theta} \times (-3qr\sin\theta)$

$= -6\tan\theta$ A1

$\int \frac{d\theta}{\tan\theta} = \int -6dt$ B1

$\ln(\sin\theta) = -6t + c$ M1 A1

Substitute given values of θ and t to get $c = \ln(\sin\frac{\pi}{6})$ B1

$\ln(\sin\theta) - \ln(\sin\frac{\pi}{6}) = -6t$

$\ln(2\sin\theta) = -6t$ M1

$2\sin\theta = e^{-6t}$ A1 [6]

(c) $2 \sin \theta = e^{-6(0.5)}$

$\sin \theta = \dfrac{e^{-3}}{2}$ B1

Area of $\Delta PQR = ½\, qr\, (\dfrac{e^{-3}}{2})$ M1

$= \dfrac{1}{4e^3} qr \text{ cm}^2$ A1

Total marks : 14 marks

7. $X \sim B(35, 0.1)$ B1
$\rightarrow X \sim P_o\,(3.5)$

$P(X \geq 4) = 1 - P(X \leq 3)$ M1

$= 1 - e^{-3.5}(1 + 3.5 + \dfrac{3.5^2}{2!} + \dfrac{3.5^3}{3!})$ M1 (for using Poisson)

$= 0.4634$ A1

Total marks : 4 marks

8. (a) Area under the curve $= \dfrac{1}{2}(5-1) \cdot k = 1$ M1

$k = ½$ A1

(b) $\dfrac{y-0}{x-1} = \dfrac{k}{2}$ or $\dfrac{y-0}{x-3} = -\dfrac{k}{2}$ M1

$y = \dfrac{1}{4}x - \dfrac{1}{4}$ $y = -\dfrac{1}{4}x + \dfrac{3}{4}$ A1A1

$$f(x) = \begin{cases} \dfrac{1}{4}x - \dfrac{1}{4}, & 1 \leq x \leq 3 \\ -\dfrac{1}{4}x + \dfrac{3}{4}, & 3 \leq x \leq 5 \\ 0, & \textit{otherwise} \end{cases}$$ B1

Total: 6 marks

9. (a) $P(A)=\frac{5}{6}\times\frac{4}{5}\times\frac{3}{4}$ OR $P(A)=\frac{\text{n(A)}}{\text{n(S)}}=\frac{5}{6}\times\frac{4}{5}\times\frac{3}{4}$ M1 (either one)

$=\frac{1}{2}$ $=\frac{1}{2}$ A1

$P(B)=\frac{1}{6}\times\frac{5}{5}\times\frac{4}{4}$

$=\frac{1}{6}$ B1

(b) $P(A\cap B)=\frac{1}{6}\times\frac{4}{5}\times\frac{3}{4}=\frac{1}{10}$ B1

$P(A|B)=\frac{1/10}{1/6}=\frac{3}{5}$ M1 A1

(c) A and B are not independent since $P(A)\neq P(A|B)$ B1B1

Total marks : 8 marks

10. Distances travelled

4	6 7 7 8
5	0 2 3 4 8
6	2 3 3 6 7 8 8
7	2 4 6 7 8
8	0 1 1 2

Key : 6 | 2 means 62 km *B1 (correct stem and leaf, and key)*
B1 (all right)

b) Median = 66 km., Q_1 = 52.5 km., Q_3=76.5 km. *B1, B1*

(OR: using $\frac{n}{4}$ and $\frac{3n}{4}$ method : Q_1 = 53 km, Q_3 = 76 km)

IQR = 24 km (or 23 km) *B1*

(c) Box Plot :

Whiskers from 46 km to 82 km. *B1*

Box : Median = 66 km., Q_1= 52.5km., Q_3= 76.5km

B1(his median and quartiles) B1 (All right)

NOTE: Other method: Median = 66 km., Q_1= 53 km., Q_3= 76 km

d) $\sum x = 1613$, $\sum x^2 = 107701$ *B1*

Mean = 1613÷25 = 64.52 km *M1A1*

Std deviation

$$= \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

$$= \sqrt{\frac{107701}{25} - \left(\frac{1613}{25}\right)^2}$$ M1

$= 12.05$ A1

e) Distribution is slightly skewed to the left or almost symmetrical. *B1*
Mean slightly less than median or mean is approximately the same as median) *B1*

Total : 15 marks

11(a) $P(X=x)=k\left|\frac{3}{2}-x\right|$

From $\sum P(X=x)=1$

$k\left|\frac{3}{2}-1\right|+k\left|\frac{3}{2}-2\right|+k\left|\frac{3}{2}-3\right|=1$ M1

$k\left(\frac{1}{2}+\frac{1}{2}+\frac{3}{2}\right)=1$

$k=\frac{2}{5}$ A1

(b)

$X=x$	1	2	3
$P(X=x)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{3}{5}$

$E(X)=\sum xP(x)$

Mean = $1\left(\frac{1}{5}\right)+2\left(\frac{1}{5}\right)+3\left(\frac{3}{5}\right)$ M1

$=\frac{1}{5}+\frac{2}{5}+\frac{9}{5}$

$=\frac{12}{5}$ A1

$E(X^2)=\sum x^2P(X=x)$

$=1\left(\frac{1}{5}\right)+4\left(\frac{1}{5}\right)+9\left(\frac{3}{5}\right)$

$=\frac{1}{5}+\frac{4}{5}+\frac{27}{5}$

$=\frac{32}{5}$ B1

$\text{Var}(X) = \frac{32}{5} - \left(\frac{12}{5}\right)^2$ M1

$= \frac{32}{5} - \frac{144}{25}$

$= \frac{16}{25}$ A1

Total: 7 marks

12. Let X be diameter of the rod

$X \sim N(2.00, \sigma^2)$

Let Y be internal diameter of the ring

$Y \sim N(2.00 + 3\sigma, (3\sigma)^2)$ B1 (for either X or Y)

$Y - X \sim N(3\sigma, 10\sigma^2)$ B1

$P(Y > X) = P(Y - X > 0)$ M1

$= P(Z > \frac{0 - 3\sigma}{\sigma\sqrt{10}})$

$= P(Z > -\frac{3}{\sqrt{10}})$ M1

$= 0.829$ (3 d.p) A1 [5]

$P(X > 2.05) = 0.01$ M1

$P(Z > \frac{0.05}{\sigma}) = 0.01$ M1

$\frac{0.05}{\sigma} = 2.326$ B1 (for 2.326)

$\sigma = 0.021$ (3 d.p) A1 [4]

Total = 9 marks