SMJK HENG EE, PULAU PINANG
PEPERIKSAAN PERCUBAAN STPM TAHUN 2010
MATHEMATICS S/T
PAPER 1
TINGKATAN ENAM ATAS

Tarikh : 24 September 2010
Masa : 8.00 – 11.00 (3 jam)

Disediakan oleh : Pn. Ooi H.B.
Disemak oleh : Pn. Khor A.N.
Cik Teh G.B. (Language)

Instruction :
Answer all questions.

1. Given the universal set $\xi = \{a,b,c,d,e,f,g\}$, $A = \{a,b,c,d,e\}$, $B = \{a,c,e,g\}$ and $C = \{b,e,f,g\}$. Find
   (i) $A \cup C$ (ii) $A' - B$ (iii) $(A - C)'$ (iv) $A \cap C'$ [4 *marks*]

2. Solve the inequality
   $\frac{15}{x^2} > 1 + \frac{2}{x}$ [4 *marks*]

3. Given that $z = x + ix$ where x is a non zero real number. Show that
   (i) $z - z^* = i(z + z^*)$ [3 *marks*]
   (ii) $\frac{1}{z^*} = i\left(\frac{1}{z}\right)$ [4 *marks*]

4. Differentiate with respect to x :
   (i) $e^{-3x} \sin 5x$ [3 marks]
   (ii) $\frac{2^x}{1 + x^3}$ [4 *marks*]

5. Find
   (i) $\int \frac{x}{e^{x+1}} dx.$ [3 *marks*]
   (ii) $\int \frac{1}{x \ln x} dx.$ [4 *marks*]

6. If $\frac{y}{x}$ is small enough for powers of $\frac{y}{x}$ higher than the third to be neglected, show that
   $(x + y)^{\frac{1}{2}} - (x - y)^{\frac{1}{2}} = x^{\frac{1}{2}}\left(\frac{y}{x} + \frac{y^3}{8x^3}\right)$. Hence, or otherwise deduce a rational approximation to $\sqrt{5} - \sqrt{3}$ and a rational approximation to $\sqrt{5} + \sqrt{3}$. [7 *marks*]

7. The matrix A is defined as $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ k & 4 & 1 \end{pmatrix}$

(i) Find the value of k if A^{-1} not exist. [2 *marks*]

(ii) If $k = -3$, and $B = A^2 - A - 7I$ where I is matrix of order 3 x 3, then find the matrix B. [3 *marks*]

Hence, show that $AB + 15I = 0$. [3 *marks*]

8. A man deposits RM5000.00 at the start of every year into his bank account . The interest paid by the bank is 4% a year. Any interest he receives is added to the principal. Find the sum of his savings in the bank at the end of the 12^{th} year (including interest received in the 12^{th} year).
If he withdraws a sum of RM30000.00 at the end of the 12^{th} year and he stops making further deposits, calculate his savings at the end of the 15^{th} year. [8 *marks*]

9. $A(0,6)$, $B(3,-2)$, $C(8,4)$ and $D(a,b)$ are vertices of a parallelogram $ABCD$.
Determine the values of a and b. [4 *marks*]
Find the shortest distance from the vertex A to the side BC and find the area of the parallelogram $ABCD$. [6 *marks*]

10. Given that the equation $ax^2 + bx + c = 0$ has roots α and β.

(i) If $\beta = 2\alpha$ and $b = a + c$, express a in terms of c. [5 *marks*]

(ii) Show that $c^3x^2 + (b^3 - 3abc)x + a^3 = 0$ has roots $\dfrac{1}{\alpha^3}$ and $\dfrac{1}{\beta^3}$. [5 *marks*]

11. The functions f and g are defined as

$$f(x) = x^2 - 1, \ x \in R$$

$$g(x) = \frac{x+1}{x-2}, \ x \in R, \ x \neq 2$$

Sketch the graph for each of these functions. [4 *marks*]

(i) Determine the inverse function g^{-1}. [3 *marks*]

(ii) Find the composite function $f \circ g$ and sketch its graph.
State the range for this composite function. [7 *marks*]

12. The base of a rectangular box has length 4 times its width and the total surface area of the box (this is a closed box) is $400\,cm^2$. Find an expression for the volume of the box in terms of its width. [4 *marks*]
Find the length, width and height of the box when its volume is maximum and find this maximum value. [8 *marks*]
Sketch the graph of the volume as a function of the width. [2 *marks*]

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1) i) $A \cup C = \xi$ 1

ii) $A' - B = \{f\}$ 1

iii) $(A-C)' = C = \{b, e, f, g\}$ 1

iv) $A \cap C' = \{a, c, d\}$ 1

2) $\frac{15}{x^2} > 1 + \frac{2}{x}$

$\frac{x^2+2x-15}{x^2} < 0$ 1

If $x \neq 0$, then $x^2 > 0$ for any real values of x 1

$x^2+2x-15 < 0$ 1

$(x+5)(x-3) < 0$

$\{x : -5 < x < 3,\ x \neq 0,\ x \in R\}$ 1

3) $z = x + ix$

i) $z - z^* = (x+ix) - (x - ix)$

$= 2ix$ 1

$z + z^* = x + ix + (x - ix)$

$= 2x$ 1

$\therefore\ z - z^* = i(z + z^*)$ 1

ii) $\frac{1}{z^*} = i\left(\frac{1}{z}\right)$

$\frac{1}{z^*} = \frac{1}{x-ix}$

$= \frac{1}{x-ix} \times \frac{i}{i}$ 1

$= \frac{i}{x+xi}$ 1

$= i\left(\frac{1}{x+xi}\right)$ 1

$= i\left(\frac{1}{z}\right)$ 1

4) i) let $y = e^{-3x}\sin 5x$

$\frac{dy}{dx} = e^{-3x}(5\cos 5x) - 3e^{-3x}\sin 5x$ 1 1

$= e^{-3x}(5\cos 5x - 3\sin 5x)$ 1

ii) let $y = \frac{2^x}{1+x^3}$

$\ln y = \ln 2^x - \ln(1+x^3)$ 1

$= x\ln 2 - \ln(1+x^3)$ 1

$\frac{1}{y}\frac{dy}{dx} = \ln 2 - \frac{3x^2}{1+x^3}$ 1

$\frac{dy}{dx} = \left[\ln 2 - \frac{3x^2}{1+x^3}\right]\left[\frac{2^x}{1+x^3}\right]$

$\frac{dy}{dx} = \left[\frac{2^x}{(1+x^3)^2}\right]\left[(1+x^3)\ln 2 - 3x^2\right]$ 1

$\frac{dy}{dx} = \frac{2^x \ln 2 (1+x^3) - 2^x(3x^2)}{(1+x^3)^2}$ 1 1 1

$= \frac{2^x}{(1+x^3)^2}\left[(1+x^3)\ln 2 - 3x^2\right]$ 1

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5) i) $\int \frac{x}{e^{x+1}}\,dx = \int x e^{-(x+1)}\,dx$

let $u = x$ $\qquad \frac{dv}{dx} = e^{-(x+1)}$

$\frac{du}{dx} = 1$ $\qquad v = -e^{-(x+1)}$

$$\int \frac{x}{e^{x+1}}\,dx = \int x\, e^{-(x+1)}\,dx$$
$$= x[-e^{-(x+1)}] - \int [-e^{-(x+1)}](1)\,dx \quad (1)$$
$$= -xe^{-(x+1)} + [e^{-(x+1)}] + c$$
$$= -e^{-(x+1)}[(x+1)] + c \quad (1)$$
$$= -\frac{(x+1)}{e^{(x+1)}} + c \quad (1)$$

ii) $\int \frac{1}{x \ln x}\,dx$ $\qquad$ let $u = \ln x$, $\frac{du}{dx} = \frac{1}{x}$

$= \int \frac{1}{x}\left(\frac{1}{u}\right) x\,du$ (1)

$= \int \frac{1}{u}\,du$ (1)

$= \ln|u| + c$ (1)

$= \ln|\ln x| + c$ (1)

or

$\int \frac{1}{x \ln x}\,dx$

$= \int \frac{\frac{1}{x}}{\ln x}\,dx$

$= \ln|\ln x| + c$ (4)

6) $(x+y)^{\frac{1}{2}} = x^{\frac{1}{2}}\left(1+\frac{y}{x}\right)^{\frac{1}{2}}$

$= x^{\frac{1}{2}}\left(1 + \frac{y}{2x} - \frac{1}{8}\frac{y^2}{x^2} + \frac{y^3}{16x^3} + \ldots\right)$ — (1) (1)

$(x-y)^{\frac{1}{2}} = x^{\frac{1}{2}}\left(1-\frac{y}{x}\right)^{\frac{1}{2}}$

$= x^{\frac{1}{2}}\left(1 - \frac{y}{2x} - \frac{y^2}{8x^2} - \frac{y^3}{16x^3} + \ldots\right)$ — (2) (1)

(1) − (2), $(x+y^{\frac{1}{2}}) - (x-y)^{\frac{1}{2}} = x^{\frac{1}{2}}\left[\frac{y}{x} + \frac{y^3}{8x^3}\right]$ (1)

Using $x = 4$, $y = 1$ then $(x+y)^{\frac{1}{2}} - (x-y)^{\frac{1}{2}} = \sqrt{5} - \sqrt{3}$

$\sqrt{5} - \sqrt{3} = 4^{\frac{1}{2}}\left[\frac{1}{4} + \frac{1}{8 \times 64}\right]$ (1)

$= 2\left(\frac{129}{512}\right)$

$= \frac{129}{256}$ (1)

$(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3}) = 2$

$(\sqrt{5}+\sqrt{3}) = 2\left(\frac{256}{129}\right)$ (1)

$= \frac{512}{129}$ (1)

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7) A^{-1} not exist when $|A| = 0$

$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ k & 4 & 1 \end{pmatrix}$

$|A| = 1(-9) - 1(1-2k) + 1(4+k)$ (1)

$0 = -9 - 1 + 2k + 4 + k$

$k = 2$ (1)

if $k = -3$

$B = A^2 - A - 7I$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & 4 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & 4 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & 4 & 1 \end{pmatrix} - 7\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (1)$$

$$= \begin{pmatrix} -1 & 4 & 4 \\ -6 & 10 & 1 \\ -2 & -3 & 6 \end{pmatrix} (1) - \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & 4 & 1 \end{pmatrix} - \begin{pmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} -9 & 3 & 3 \\ -7 & 4 & -1 \\ 1 & -7 & -2 \end{pmatrix} \quad (1)$$

$$AB = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ -3 & 4 & 1 \end{pmatrix}\begin{pmatrix} -9 & 3 & 3 \\ -7 & 4 & -1 \\ 1 & -7 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} -15 & 0 & 0 \\ 0 & -15 & 0 \\ 0 & 0 & -15 \end{pmatrix} \quad (1)$$

$$= -15\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (1)$$

$= -15I$

$AB + 15I = 0$ (1)

8) Total savings at the end of the 1^{st} year $= 5000 + \frac{4}{100}(5000)$ (1)

$= 5000(1.04)$ (1)

Total savings at the end of the 2^{nd} year $= [5000 + 5000(1.04)](1.04)$

$= 5000(1.04) + 5000(1.04)^2$

Total savings at the end of the 3^{rd} year $= [5000(1.04) + 5000(1.04)^2](1.04)$

$= 5000[1.04 + 1.04^2 + 1.04^3]$ (1)

Total saving at the end of 12^{th} year $= 5000[1.04 + 1.04^2 + \ldots + (1.04)^{12}]$ (1)

$= 5000(1.04)\left[\frac{1.04^{12} - 1}{1.04 - 1}\right]$

$= RM78134.19$ (1)

The man withdraws RM30000 at the end of the 12^{th} year

The balance at the end of the 12^{th} year $= RM78134.19 - RM30000$ (1)

$= RM48134.19$ (1)

Total savings at the end of the 15^{th} year $= 48134.19(1.04)^3$

$= RM54144.42$ (1)

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9) mid-point of BD = mid-point of AC

$\frac{a+3}{2} = \frac{0+8}{2}$ (1)

$a = 5$ (1)

$\frac{b-2}{2} = \frac{6+4}{2}$ (1)

$b - 2 = 10$

$b = 12$ (1)

Equation of BC:

$[y-(-2)] = \frac{4-(-2)}{8-3}(x-3)$ (1)

$y+2 = \frac{6}{5}(x-3)$

$5y+10 = 6x-18$

$6x-5y-28=0$ (1)

The shortest distance from A to BC

$d = \frac{|6(0)-5(6)-28|}{\sqrt{6^2+5^2}}$ (1)

$= \frac{58}{\sqrt{61}}$ (1)

$BC = \sqrt{(8-3)^2+(4+2)^2}$

$= \sqrt{61}$

Area of the parallelogram ABCD $= (BC)(d)$

$= \sqrt{61}\left(\frac{58}{\sqrt{61}}\right)$ (1)

$= 58 \text{ unit}^2$ (1)

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10) i) $\alpha+\beta=-\frac{b}{a}$ — ① $\alpha\beta=\frac{c}{a}$ — ② (1)

$\beta=2\alpha$

$3\alpha=-\frac{b}{a}$ $2\alpha^2=\frac{c}{a}$ — ④ } (1)

$\alpha=-\frac{b}{3a}$ — ③

From ③ $\alpha=-\frac{b}{3a}$ substitute into ④ $2\left[-\frac{b}{3a}\right]^2=\frac{c}{a}$

$2\left(\frac{b^2}{9a^2}\right)=\frac{c}{a}$

$2b^2=9ac$ (1)

Given $b=a+c$ $9ac=2(a+c)^2$

$9ac=2a^2+4ac+2c^2$ } (1)

$2a^2-5ac+2c^2=0$

$(2a-c)(a-2c)=0$

$a=\frac{c}{2}$ or $a=2c$ (1)

ii) $\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\alpha^3+\beta^3}{(\alpha\beta)^3}$

$$=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^3}$$

$$=\frac{-\frac{b^3}{a^3}-3\left(\frac{c}{a}\right)\left(-\frac{b}{a}\right)}{\left(\frac{c}{a}\right)^3} \quad (1)$$

$$=\frac{-b^3+3abc}{c^3} \quad (1)$$

$$\left(\frac{1}{\alpha^3}\right)\left(\frac{1}{\beta^3}\right)=\frac{1}{(\alpha\beta)^3}$$

$$=\frac{1}{\left(\frac{c}{a}\right)^3}$$

$$=\left(\frac{a}{c}\right)^3 \quad (1)$$

The quadratic equation is $x^2-\left[\frac{-b^3+3abc}{c^3}\right]x+\frac{a^3}{c^3}=0$ (1)

$c^3x^2+(b^3-3abc)x+a^3=0$ (1)

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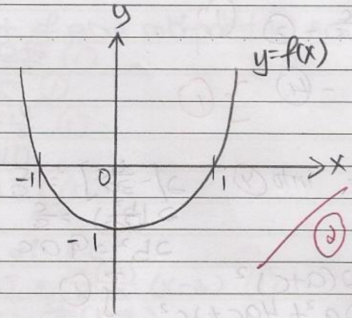
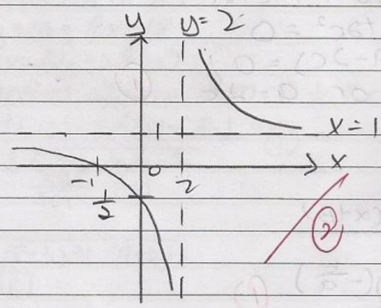
11)





i) let $g^{-1}(x)=y$

$g(y)=x$

$x=\frac{y+1}{y-2}$

$x(y-2)=y+1$

$xy-2x-y-1=0$

$y(x-1)-2x-1=0$

$y=\frac{2x+1}{x-1}$

$g^{-1}(x)=\frac{2x+1}{x-1},\ x\neq 1, x\in R$

$$f\circ g(x)=f\left[\frac{x+1}{x-2}\right]$$

$$=\left[\frac{x+1}{x-2}\right]^2-1$$

$$=\frac{x^2+2x+1}{x^2-4x+4}-1$$

$$=\frac{6x-3}{x^2-4x+4},\ x\neq 2, x\in R$$

let $y=\frac{6x-3}{x^2-4x+4}$

$$\frac{dy}{dx}=\frac{(x^2-4x+4)(6)-(6x-3)(2x-4)}{(x^2-4x+4)^2}$$

$$=\frac{-6x^2+6x+12}{(x^2-4x+4)^2}$$

$\frac{dy}{dx}=0$ at turning point

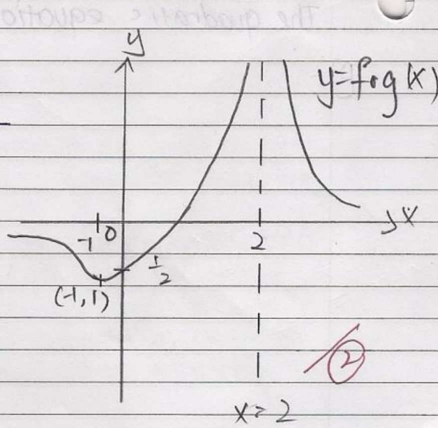
$-6x^2+6x+12=0$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2,\ x=-1$

when $x=-1$, $y=-1$



Range of fog is $[-1,\infty)$

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12) let the width of the box be x cm

Surface area of the box

$= 2xh + 2(4x)x + 2(4xh)$

$= 10xh + 8x^2$

$10xh + 8x^2 = 400$ (1)

$h = \frac{400 - 8x^2}{10x}$

$= \frac{40}{x} - \frac{4}{5}x$ (1)

Volume of the box $= x(4x)h$

$= 4x^2\left(\frac{400-8x^2}{10x}\right)$ (1)

$= \frac{2}{5}x(400 - 8x^2)$

$= 160x - \frac{16}{5}x^3$ (1)

$V = 160x - \frac{16}{5}x^3$

$\frac{dV}{dx} = 160 - \frac{48}{5}x^2$ (1)

let $\frac{dV}{dx} = 0$ $\quad 160 - \frac{48}{5}x^2 = 0$ (1)

$x = \frac{5\sqrt{2}}{\sqrt{3}}$

$x = \frac{5}{3}\sqrt{6}$ cm (1)

$\frac{d^2V}{dx^2} = -\frac{96}{5}x$

when $x = \frac{5}{3}\sqrt{6}$

$\frac{d^2V}{dx^2} = -78.38 < 0$

(max value)

So, V attains its maximum when $x = \frac{5}{3}\sqrt{6}$ (1)

$h = \frac{400 - 8\cdot\frac{50}{3}}{10\cdot\frac{5}{3}\sqrt{6}}$

$h = \frac{16}{\sqrt{6}}$

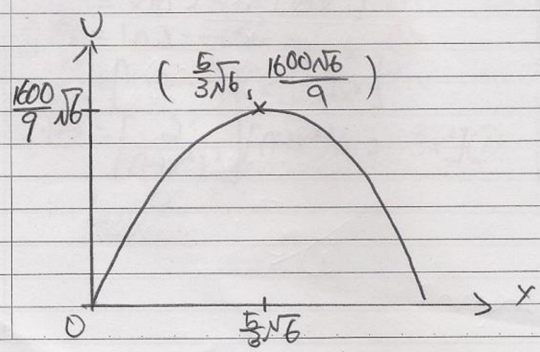
$= \frac{8}{3}\sqrt{6}$

Maximum volume

$= \frac{2}{5} \times \frac{5}{3}\sqrt{6} \times \left[400 - 8\cdot\frac{50}{3}\right]$

$= \frac{1600}{9}\sqrt{6}$ or 435.64 cm^3 (1)

width $= \frac{5}{3}\sqrt{6}$ (1) length $= \frac{20}{3}\sqrt{6}$ (1) height $= \frac{8}{3}\sqrt{6}$ (1)

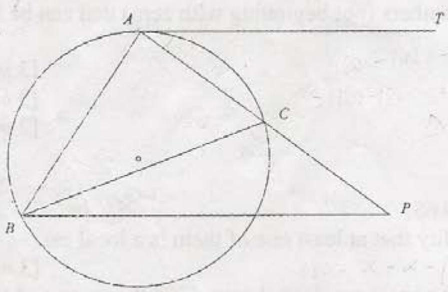


SMJK HENG EE, PULAU PINANG
PEPERIKSAAN PERCUBAAN STPM / 2010
MATHEMATICS T
PAPER 2
TINGKATAN ENAM ATAS

Tarikh: 28 Sept 2010 | Disediakan oleh: Pn. Khor AN
Masa : 8.00 – 11.00 (3 jam) | Disemak oleh: Pn. Ooi BL
En.Wong KC (Language)

Instructions:
*Answer **all** questions.*

1. A boat travelling at a constant rate due east passes a buoy that is 1.0 kilometer from a lighthouse. The lighthouse is N30°E of the buoy. After the boat has travelled for half an hour, its bearing to the lighthouse is N74°W. How fast is the boat travelling? [4 *marks*]

2. 

AT is a tangent to the circle at A. BP is parallel to AT and ACP is a straight line. Show that $AP \cdot AC = AB^2$. [7 *marks*]

3. Two particles A and B move freely on a plane with velocities $-3\mathbf{i} + 29\mathbf{j}$ ms^{-1} and $3\mathbf{i} + 21\mathbf{j}$ ms^{-1} respectively.
   (a) Find the velocity of B relative to A and also the vector $\overrightarrow{AB}$ at time t, given that when $t = 0$, $\overrightarrow{AB} = -56\mathbf{i} + 8\mathbf{j}$ m. [3 *marks*]
   (b) Show that the distance between A and B is the shortest when $t = 4$ s, and find this distance. [5 *marks*]

4. Given $\overrightarrow{AB} = m\mathbf{i} + 8\mathbf{j}$ and $\overrightarrow{CB} = 18\mathbf{i} - 4\mathbf{j}$, where m is a constant. Find the values of m if
   (a) $|\overrightarrow{AC}| = 20$ units, [6 *marks*]
   (b) A, B and C are collinear. [3 *marks*]

5. Given two vectors $\mathbf{a}$ and $\mathbf{b}$ ($\mathbf{a} \neq 0$, $\mathbf{b} \neq 0$), show that
   (a) if $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ are perpendicular, then $|\mathbf{a}| = |\mathbf{b}|$, [4 *marks*]
   (b) if $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$ then $\mathbf{a}$ and $\mathbf{b}$ are perpendicular to each other. [5 *marks*]

6. (a) If $\tan\theta = \dfrac{3}{4}$ and θ is acute, find the values of $\tan 2\theta$, $\tan 4\theta$ and $\tan\dfrac{\theta}{2}$. [8 *marks*]
   (b) Express $\sqrt{\dfrac{1 - \sin 2\theta}{1 + \sin 2\theta}}$ in terms of $\tan\theta$. [4 *marks*]

7. The probability distribution of a discrete random variable X is given by $P(X = x) = (1 - p)^{x-1} p$ for $x = 1, 2, 3, \ldots$ and $0 < p < 1$. Show that $E(X) = \frac{1}{p}$. [5 *marks*]

8. If $X \sim N(50, 9)$, find
   (a) $P(51 < X < 54)$, [3 *marks*]
   (b) the value of k if $P(X > k) = 0.2543$. [4 *marks*]

9. (a) If events A and B are independent, show that events $\overline{A}$ and B are independent. [4 *marks*]
   (b) X and Y are two events such that $P(X) = \frac{1}{2}$ and $P(Y) = \frac{1}{4}$. Given that events X and Y are mutually exclusive, find $P(X \cup Y)$ and $P(Y \cap \overline{X})$. [3 *marks*]

10. Find the number of different 7-digit telephone numbers (not beginning with zero) that can be formed using the digits 0 through 9 without repetition if
   (a) the number is an even number, [3 *marks*]
   (b) the number is a multiple of 5, [3 *marks*]
   (c) the first three digits are 987 in that order. [2 *marks*]

11. The probability that a person buys a local car is 0.65.
   (a) If Mr. Lee has 3 cars, what is the probability that at least one of them is a local car. [3 *marks*]
   (b) There are 50 families who own 3 cars in a certain residential area. Find the mean and the standard deviation of the number of families that own at least one local car. [5 *marks*]

12. In an agricultural experiment, the increase in the masses of 100 sheep in a certain period are recorded below:

Increase in mass (kg)	Frequency
4 – 8	3
9 – 13	8
14 – 18	25
19 – 23	34
24 – 28	15
29 – 33	12
34 – 38	3

   (a) Calculate the mean and the standard deviation of the increase in the mass of the sheep. [5 *marks*]
   (b) Plot a cumulative frequency curve of the above data. Hence, estimate the median and the semi interquartile range. [8 *marks*]
   (c) Find the percentage of the sheep with masses is at least 30 kg. [3 *marks*]